AP Precalculus

UNIT 1A - Rates of Change, Polynomial Functions

ESSENTIAL QUESTION	BIG IDEAS
How can rates of change and polynomial functions provide students with vital tools for understanding the world?	Students will describe how input/output values of a function vary together by comparing function values, and how two quantities vary together at different points and over different intervals. Students will construct a graph representing two quantities that vary with respect to each other in a contextual scenario. Students will calculate average rates of change at two points using average rates of change near the points; students will calculate average rates of change for sequences and specific function types, and students will determine the change in the average rates of change for specific function types. Students will identify key characteristics of polynomial functions related to rates of change, zeros, symmetry, and end behavior.

GUIDING QUESTIONS

Content and Process

- What is the relationship between a function and its domain and range? **1.1.A.1**
- What is the relationship between a function's concavity, rate of change, and points of inflection? **1.1.B.3, 1.1.B.4, 1.3.B.2, 1.3.B.3, 1.4.A.5**
- What information can be gleaned from an average rate of change, the sign of an average of a change, and the average rate of change at a point? **1.2.A.1, 1.2.A.2, 1.2.A.3, 1.2.B.1, 1.2.B.2, 1.2.B.3**
- What does the average rate of change, and the change in the average rates of change for linear, quadratic, and other function types tell us? **1.3.A.1, 1.3.A.2, 1.3.A.3, 1.3.B., 1.3.B.2, 1.3.B.3**
- When do local (relative) or global (absolute) extrema occur? **1.4.A.2, 1.4.A.3, 1.4.A.4**
- What is the relationship between the terms zero, root, x-intercept, and linear factor? **1.1.B.5, 1.5.A.3**
- What is the relationship between the total number of zeros for a polynomial, multiplicity, and complex zeros? **1.5.A.1, 1.5.A.2, 1.5.A.3, 1.5.A.4, 1.5.A.5**
- What does it mean for a function to be odd or even both graphically and analytically? **1.5.B.1, 1.5.B.2**
- How can the end behavior of a polynomial function based on its equation be determined? **1.6.A.1, 1.6.A.2, 1.6.A.3**

***The indicators listed above correspond to the *AP Precalculus Course and Exam Booklet*.

Reflective

- How do I use graphical, numerical, analytical, and verbal representations to answer questions or construct models?
- How do I describe characteristics of a function?
- How do I support conclusions or choices with a logical rationale or appropriate data?
- How do I express functions, equations, or expressions in analytically equivalent forms as most appropriate in a particular context?

FOCUS STANDARDS

Standards of Mathematical Practice

1.B. Express functions, equations, or expressions in analytically equivalent forms that are useful in a given mathematical or applied context.
2.A Identify information from graphical, numerical, analytical, and verbal representations to answer a question or construct a model, with and without technology.
2.B Construct equivalent graphical, numerical, analytical, and verbal representations of functions that are useful in a given mathematical or applied context, with and without technology.
3.A Describe the characteristics of a function with varying levels of precision, depending on the function representation and available mathematical tools.
3.B Apply numerical results in a given mathematical or applied context.
3.C Support conclusions or choices with a logical rationale or appropriate data.

Content Standards
F.IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y=f(x)$.
F.IF.4 For a function that models a relationship between two quantities, interpret key features of expressions, graphs and tables in terms of the quantities, and sketch graphs showing key features given a description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.
F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. (9/10) limited to linear functions.
F.IF.9 Compare properties of two functions using a variety of representations (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, a quantity increasing exponentially eventually exceeds a quantity increasing linearly.

UNIT 1B - Rational Functions

ESSENTIAL QUESTION	BIG IDEAS
How can rational functions and modeling provide students with vital tools for understanding the world?	Students will analyze graphs and equations of rational functions with specific attention paid to end behavior, zeros, vertical asymptotes, and holes. Students will rewrite polynomial and rational expressions in equivalent forms, including determining the quotient of two polynomial functions with long division, and expanding binomials with the binomial theorem. Students will construct function types that are additive and/or multiplicative transformations of other functions. Students will identify an appropriate function type to construct a model (linear, quadratic, cubic, quartic, polynomial of degree n, or piecewise-defined, rational) for a given scenario, will describe assumptions and restrictions related to their model, and will apply their model to answer questions about a data set or contextual scenario.

GUIDING QUESTIONS

Content and Process

- How is the end behavior of a rational function both determined and expressed? **1.7.A.1, 1.7.A.2, 1.7.A.3, 1.7.A.4, 1.7.A.5, 1.7.A.6**
- How are zeros of rational functions determined and expressed? **1.8.A.1, 1.8.A.2**
- How are vertical asymptotes of rational functions both determined and expressed? **1.9.A.1, 1.9.A.2**
- How are holes of rational functions both determined and expressed? **1.10.A.1, 1.10.A.2**
- In analyzing rational functions and polynomials, why is it sometimes advantageous for the function to be in factored form and why is it sometimes advantageous for the function to be in standard form? **1.11.A.1, 1.11.A.2, 1.11.A.3**
- What is the relationship between polynomial long division and graphs of rational functions? **1.11.B.1, 1.11.B.2**
- What is the role of the binomial theorem in expanding polynomial functions? **1.11.C.1**
- How do different types of transformations alter the graph of a parent function and its domain and range? **1.12.A.1, 1.12.A.2, 1.12.A.3, 1.12.A.4, 1.12.A.5, 1.12.A.6**
- When is it appropriate to model data sets or aspects of contextual scenarios with linear functions, quadratic functions, polynomial functions, or piecewise-defined functions? **1.13.A.1, 1.13.A.2, 1.13.A.3, 1.13.A.4, 1.13.A.5, 1.13.A.6, 1.13.A.7**
- What are the roles of assumptions and restrictions when building a function model? **1.13.B.1, 1.13.B.2, 1.13.B.3, 1.13.B.4**
- How are different function models constructed and how can function models answer question questions about a data set or contextual scenario? **1.14.A.1, 1.14.A.2, 1.14.A.3, 1.14.A.4, 1.14.B, 1.14.C.1**

***The indicators listed above correspond to the *AP Precalculus Course and Exam Booklet*.

Reflective

- How do I solve equations analytically?
- How do I express functions, equations, or expressions in analytically equivalent forms as most appropriate in a particular context?
- How do I construct new functions with transformations and regressions and what information can be obtained?
- How do I identify information from different representations to construct a model?
- How do I describe characteristics of a rational function?
- How do I apply numerical results in a given mathematical or applied context?
- How do I support conclusions or choices with a logical rationale or appropriate data?

FOCUS STANDARDS

Standards of Mathematical Practice
1.A. Solve equations and inequalities represented analytically, with and without technology.
1.B. Express functions, equations, or expressions in analytically equivalent forms that are useful in a given mathematical or applied context.
1.C. Construct new functions, using transformations, compositions, inverses, or regressions, that may be useful in modeling contexts, criteria, or data, with and without technology.
2.A Identify information from graphical, numerical, analytical, and verbal representations to answer a question or construct a model, with and without technology.
3.A Describe the characteristics of a function with varying levels of precision, depending on the function representation and available mathematical tools.
3.B Apply numerical results in a given mathematical or applied context.
3.C Support conclusions or choices with a logical rationale or appropriate data.

Content Standards
A.APR.5. (+) Know and apply the Binomial Theorem for the expansion of in powers of *x* and *y* for a positive integer *n*, where *x* and *y* are any numbers, with coefficients determined for example by Pascal's Triangle. The Binomial Theorem can be proven by mathematical induction or by a combinatorial argument.
A.APR.6. (+) Rewrite simple rational expressions in different forms; write $\frac{a(x)}{b(x)}$ in the form $q(x)+\frac{r(x)}{b(x)}$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.
F.IF.4. (all) For a function that models a relationship between two quantities, interpret key features of expressions, graphs and tables in terms of the quantities, and sketch graphs showing key features given a description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity
F.IF.5. (all) Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a*

factory, then the positive integers would be an appropriate domain for the function.
F.IF.7e. (11) Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
F.IF.7f. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
F.BF.3. Transform parent functions $(f(x))$by replacing $f(x)$with $f(x)+k, kf(x), f(kx), and\ f(x+k)$for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.
F.LQE.1. Distinguish between situations that can be modeled with linear functions and with exponential functions

UNIT 2A - Exponential Functions

ESSENTIAL QUESTION	BIG IDEAS
How are exponential function models widespread in the natural and social sciences?	Students will construct formulas for and describe the relationships between arithmetic sequences, geometric sequences, linear equations, and exponentia equations. Students will construct and interpret models that represent situations involvinç proportional output values over equal-length input-value intervals, and use these models to answer questions about a data set or contextual scenario. Students will construct linear, quadratic, and exponential models based on a data set and validate their selection. Students will describe the relationship between functions and their compositions, functions and their inverses, and the composition of a function and its inverse.

GUIDING QUESTIONS

Content and Process

- How can the explicit representation of both arithmetic and geometric sequences be determined, and how are these sequences similar and different? **2.1.A.1, 2.1.A.2, 2.1.A.3, 2.1.B.1, 2.1.B.2, 2.1.B.3**
- How can functions of real numbers that are comparable to arithmetic and geometric sequences be constructed, and how do arithmetic and geometric sequences relate to linear and exponential functions, respectively ? **2.2.A.1, 2.2.A.2, 2.2.A.3, 2.2.A.4, 2.2.A.5**
- How are linear and exponential functions both similar and different? **2.2.B.1, 2.2.B.2, 2.2.B.3**
- What are the key characteristics of exponential functions and how do these characteristics be displayed in graphs? **2.3.A.1, 2.3.A.2, 2.3.A.3, 2.3.A.4, 2.3.A.5**

- How can exponential expressions be rewritten in equivalent forms and what do these equivalent forms reveal? **2.4.A.1, 2.4.A.2, 2.4.A.3, 2.4.A.4**
- When is it appropriate to use an exponential function to model a contextual or mathematical scenario and how can these models be constructed, and what information does this model reveal? **2.5.A.1, 2.5.A.2, 2.5.A.3, 2.5.A.4, 2.5.A.5, 2.5.A.6, 2.5.B.1, 2.5.B.2, 2.5.B.3**
- Given sets of data, how are linear, quadratic, and exponential models constructed? **2.6.A.1, 2.6.A.2**
- How can the selection of a linear, quadratic, or exponential model be validated? **2.6.B.1, 2.6.B.2**
- How can the composition of two more functions at given values be evaluated? **2.7.A.1, 2.7.A.2, 2.7.A.3, 2.7.A.4**
- How can a representation of two or more functions be constructed and how can a given function as a composition of two or more functions be rewritten? **2.7.B.1, 2.7.B.2, 2.7.B.3, 2.7.C.1, 2.7.C.2, 2.7.C.3**
- What is an inverse function and how is it determined? **2.8.A.1, 2.8.A.2, 2.8.B.1, 2.8.B.2, 2.8.B.3, 2.8.B.4, 2.8.B.5**

***The indicators listed above correspond to the *AP Precalculus Course and Exam Booklet*.

Reflective

- How does computing an inverse function relate to solving an equation?
- Why might it sometimes be in our best interest to rewrite an exponential function or expression in an analytically equivalent form?
- How do I construct new functions using compositions, inverses, or regressions?
- How do I use information from graphical, numerical, analytical, and verbal representations to answer a question or to construct a linear, quadratic, or exponential model?
- How do I describe characteristics of linear and exponential functions?
- How do I apply numerical results in a mathematical or applied context?
- How do I support conclusions or choices with a logical rationale or appropriate data?

FOCUS STANDARDS

Standards of Mathematical Practice

1.A. Solve equations and inequalities represented analytically, with and without technology.
1.B. Express functions, equations, or expressions in analytically equivalent forms that are useful in a given mathematical or applied context.
1.C. Construct new functions, using transformations, compositions, inverses, or regressions, that may be useful in modeling contexts, criteria, or data, with and without technology.
2.A Identify information from graphical, numerical, analytical, and verbal representations to answer a question or construct a model, with and without technology.
2.B. Construct equivalent graphical, numerical, analytical, and verbal representations of functions that are useful in a given mathematical or applied context, with and without technology.
3.A Describe the characteristics of a function with varying levels of precision, depending on the function representation and available mathematical tools.
3.B Apply numerical results in a given mathematical or applied context.
3.C Support conclusions or choices with a logical rationale or appropriate data.

Content Standards

A.SSE.3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression

F.BF.1c. Compose functions. *For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time*.

F.BF.2. Write arithmetic and geometric sequences and series both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

F.BF.4 Find inverse functions.

F.LQE.1. Distinguish between situations that can be modeled with linear functions and with exponential functions.

F.LQE.2. Construct exponential functions, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table)

S.ID.5. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.

UNIT 2B - Logarithmic Functions

ESSENTIAL QUESTION	BIG IDEAS
How are logarithmic function models widespread in the natural and social sciences?	Students will describe the relationship between exponential and logarithmic functions. Students will apply the properties of logarithms to rewrite logarithmic expressions in forms applicable to a given context. Students will develop logarithmic models in applicable situations and apply these models to answer questions about data sets. Students will describe the appropriateness of an exponential model.

GUIDING QUESTIONS

Content and Process

- What does the value of a logarithmic expression tell us? **2.9.A.1, 2.9.A.2, 2.9.A.3**
- What is the relationship between exponential and logarithmic functions and what does this tell us about the graph of a logarithmic function? **2.10.A.1, 2.10.A.2, 2.10.A.3, 2.10.A.4, 2.10.A.5**
- What are the key characteristics of logarithmic functions? **2.11.A.1, 2.11.A2, 2.11.A.3, 2.11.A.4**
- How can logarithmic expressions be rewritten in equivalent forms? **2.12.A.1, 2.12.A.2, 2.12.A.3,**

2.12.A.4

- How can exponential and logarithmic equations and inequalities be solved? **2.13.A.1, 2.13.A.2, 2.13.A.3**
- How are inverse functions for exponential and logarithmic functions determined? **2.13.B.1, 2.13.B.2**
- How can a logarithmic function model be constructed? **2.14.A.1, 2.14.A.2, 2.14.A3, 2.14.A.4, 2.14A.5, 2.14.A.6**
- What does a semi-log plot of a data set tell us about a particular model? **2.15.A.1, 2.15.A2**
- Why is it beneficial to construct a linearization of exponential data? **2.15.B.1, 2.15.B2**

***The indicators listed above correspond to the *AP Precalculus Course and Exam Booklet*.

Reflective

- How are exponential and logarithmic equations and inequalities solved?
- Why might it sometimes be in our best interest to rewrite an exponential function or expression in an analytically equivalent form?
- How do I construct new functions using inverses or regressions?
- How do I leverage different mathematical representations in mathematical and applied contexts?
- How do I describe characteristics of logarithmic functions?
- How do I apply numerical results in a mathematical or applied context?
- How do I support conclusions or choices with a logical rationale or appropriate data?

FOCUS STANDARDS

Standards of Mathematical Practice

1.A. Solve equations and inequalities represented analytically, with and without technology.
1.B. Express functions, equations, or expressions in analytically equivalent forms that are useful in a given mathematical or applied context.
1.C. Construct new functions, using transformations, compositions, inverses, or regressions, that may be useful in modeling contexts, criteria, or data, with and without technology.
2.B. Construct equivalent graphical, numerical, analytical, and verbal representations of functions that are useful in a given mathematical or applied context, with and without technology.
3.A Describe the characteristics of a function with varying levels of precision, depending on the function representation and available mathematical tools.
3.B Apply numerical results in a given mathematical or applied context.
3.C Support conclusions or choices with a logical rationale or appropriate data.

Content Standards

A.REI.3b Solve exponential and logarithmic equations.

F.IF.7.b Graph square root, cube root, and exponential functions.

F.IF.7.c Graph logarithmic functions, emphasizing the inverse relationship with exponentials and showing intercepts and end behavior.

F.BF.4 Find inverse functions.

F.BF.5 Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

UNIT 3A - Trigonometric Functions

ESSENTIAL QUESTION	BIG IDEAS
How does right triangle trigonometry from geometry relate to trigonometric functions?	Students will explain periodic phenomena. Students will describe the relationship between the unit circle, trigonometric values, and graphs of sinusoids.
How can sinusoids be used to provide students with vital tools for understanding the world?	Students will describe key characteristics of sinusoids and use sinusoids to model periodic phenomena.

GUIDING QUESTIONS

Content and Process

- How can verbal relationships of periodic functions be represented graphically? **3.1.A.1, 3.1.A.2**
- What are the key characteristics of periodic functions? **3.1.B.1, 3.1.B.2, 3.1.B.3**
- How are the sine, cosine, and tangent of angle determined using the unit circle? **3.2.A.1, 3.2.A.2, 3.2.A.3, 3.2.A.4, 3.2.A.5**
- How are the coordinates of points on a circle centered at the origin determined? **3.3.A.1, 3.3.A.2**
- How can we represent the sine and cosine functions using the unit circle? **3.4.A.1, 3.4.A.2, 3.4.A.3, 3.4.A.4**
- What are the key characteristics of sine and cosine functions? **3.5.A.1, 3.5.A.2, 3.5.A.3, 3.5.A.4, 3.5.A.5, 3.5.A.6**
- How do we identify the amplitude, vertical shift, period, and phase shift of a sinusoidal function? **3.6.A.1, 3.6.A.2, 3.6.A.3, 3.6.A.4, 3.6.A.5, 3.6.A.6**
- How do we construct sinusoidal function models of periodic phenomena? **3.7.A.1, 3.7.A.2, 3.7.A.3, 3.7.A.4, 3.7.A.5**

***The indicators listed above correspond to the *AP Precalculus Course and Exam Booklet*.

Reflective

- When is it appropriate to use a sinusoid to model data?
- How do I identify information from graphical, numerical, analytical, or verbal representations to answer questions?
- How do I leverage different mathematical representations in mathematical and applied contexts?
- How do I describe characteristics of sinusoids?
- How do I apply unit circle values and in what contexts?
- How do I support conclusions or choices with a logical rationale or appropriate data?

FOCUS STANDARDS

Standards of Mathematical Practice

1.C. Construct new functions, using transformations, compositions, inverses, or regressions, that may be useful in modeling contexts, criteria, or data, with and without technology.
2.A. Identify information from graphical, numerical, analytical, and verbal representations to answer a question or construct a model, with and without technology.
2.B. Construct equivalent graphical, numerical, analytical, and verbal representations of functions that are useful in a given mathematical or applied context, with and without technology.
3.A Describe the characteristics of a function with varying levels of precision, depending on the function representation and available mathematical tools.
3.B Apply numerical results in a given mathematical or applied context.
3.C Support conclusions or choices with a logical rationale or appropriate data.

Content Standards

F.IF.4 - For a function that models a relationship between two quantities, interpret key features of expressions, graphs and tables in terms of the quantities, and sketch graphs showing key features given a description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.
F.IF.7g - Graph trigonometric functions, showing period, midline, and amplitude.
F.TF.1 - Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
F.TF.2 - Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
F.TF.3 - Use special triangles to determine geometrically the values of sine, cosine, tangent for $\frac{\pi}{3}, \frac{\pi}{4}, and\ \frac{\pi}{6}$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi - x, \pi + x, and\ 2\pi - x$ in terms of their values for *x*, where *x* is any real number.
F.TF.4 - Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.
F.TF.5 - Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.

F.TF.6 - Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.

UNIT 3B - Trigonometric Functions Part II & Polar Functions

ESSENTIAL QUESTION	BIG IDEAS
How do we model aspects of circular and spinning objects without using the x-y plane?	Students will describe key characteristics of the tangent, secant, cotangent, and cosecant functions. Students will analytically and graphically represent the inverse sine, inverse cosine, and inverse tangent functions over a restricted domain. Students will determine which form of a trigonometric expression is most applicable in a given analytical context, and will use these forms to solve equations and inequalities. Students will describe the relationship between polar coordinates and rectangular coordinates, and will use this relationship to both graph and describe characteristics of polar functions.

GUIDING QUESTIONS

Content and Process

- How do we construct representations of the tangent function using the unit circle? **3.8.A.1, 3.8.A.2**
- What are the key characteristics of the tangent function? **3.8.B.1, 3.8.B.2, 3.8.B.3**
- How do we describe additive and multiplicative transformations involving the tangent function? **3.8.C.1 3.8.C.2, 3.8.C.3, 3.8.C.4, 3.8.C.5**
- How do we construct analytical and graphical representations of the inverse of the sine, cosine, anc tangent functions over restricted domains? **3.9.A.1, 3.9.A.2, 3.9.A.3**
- How do we solve equations and inequalities involving trigonometric functions? **3.10.A.1, 3.10.A.2 3.10.A.3**
- What are the key characteristics of the functions that involve quotients of the sine and cosine functions **3.11.A.1, 3.11.A.2, 3.11.A.3, 3.11.A.4, 3.11.A.5**
- How do we rewrite trigonometric expressions in equivalent forms with the Pythagorean identity **3.12.A.1, 3.12.A.2**
- How do we rewrite trigonometric expressions in equivalent forms with sine and cosine sum identities **3.12.B.1, 3.12.B.2, 3.12.B.3, 3.12.B.4**
- How do we solve equations using equivalent analytic representations of trigonometric functions **3.12.C.1, 3.12.C.2**

- How do we determine the location of a point in the plane using both rectangular and polar coordinates? **3.13.A.1, 3.13.A.2, 3.13.A.3, 3.13.A.4**
- How are graphs of polar functions constructed? **3.14.A.1, 3.14.A.2, 3.14.A.3**
- How can we describe the characteristics of the graph of a polar function? **3.15.A.1, 3.15.A.2, 3.15.A.3, 3.15.A.4, 3.15.A.5**

***The indicators listed above correspond to the *AP Precalculus Course and Exam Booklet*.

Reflective

- How do we solve trigonometric equations and inequalities?
- How can rewriting trigonometric expressions in different forms be beneficial?
- How can we construct the tangent, secant, cotangent, cosecant, inverse sine, inverse cosine, and inverse tangent functions both with and without technology?
- How can we identify information from a rectangular form of an equation to help us graph the polar form of the corresponding equation?
- How can we describe the characteristics of graphs learned in this unit with varying levels of precision?
- How do we apply numerical results in a given mathematical context?
- How do we support conclusions or choices with a logical rationale?

FOCUS STANDARDS

Standards of Mathematical Practice

1.A. Solve equations and inequalities represented analytically, with and without technology.
1.B. Express functions, equations, or expressions in analytically equivalent forms that are useful in a given mathematical or applied context.
1.C. Construct new functions, using transformations, compositions, inverses, or regressions, that may be useful in modeling contexts, criteria, or data, with and without technology.
2.A. Identify information from graphical, numerical, analytical, and verbal representations to answer a question or construct a model, with and without technology.
2.B. Construct equivalent graphical, numerical, analytical, and verbal representations of functions that are useful in a given mathematical or applied context, with and without technology.
3.A Describe the characteristics of a function with varying levels of precision, depending on the function representation and available mathematical tools.
3.B Apply numerical results in a given mathematical or applied context.
3.C Support conclusions or choices with a logical rationale or appropriate data.

Content Standards

F.IF.4 - Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.
F.TF.7 - Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.
F.TF.8 - Prove the Pythagorean identity

$sin^2(\theta) + cos^2(\theta) = 1$ and use it to find $sin(\theta), cos(\theta), or\ tan(\theta)$ given $sin(\theta), cos(\theta), or\ tan(\theta)$ and the quadrant.

F.TF.9 - Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

UNIT 4A - Functions Involving Parameters

[NOTE: College Board has designated this as an optional unit; this unit is not tested on the AP exam. However, there are a few topics that must be covered for JCCC credit.]

ESSENTIAL QUESTION	BIG IDEAS
How can we analyze the vertical and horizontal aspects of motion independently?	Students will describe key characteristics of a parametric planar motion relation that are related to position, direction, and rate of change. Students will describe how two quantities in an implicitly-defined relation vary together. Students will create parametric representations of conic sections and curves.

GUIDING QUESTIONS

Content and Process

- How do we construct a graph or table of values for a parametric function represented analytically? **4.1.A.1, 4.1.A.2, 4.1.A.3, 4.1.A.4, 4.1.A.5**
- What are the key characteristics of a parametric planar motion function that are related to position? **4.2.A.1, 4.2A.2, 4.2.A.3**
- What are the key characteristics of a parametric planar motion function that are related to direction and rate of change? **4.3.A.1, 4.3.A.2, 4.3.A.3, 4.3.A.4**
- How can we express motion around a circle or along a line segment parametrically? **4.4.A.1, 4.4.A.2, 4.4.A.3**
- How can we construct a graph of an equation involving two variables? **4.5.A.1, 4.5.A.2, 4.5.A.3**
- How can we determine how two quantities related in an implicitly defined function vary together? **4.5.B.1, 4.5.B.2**
- How can we represent conic sections with horizontal or vertical symmetry analytically? **4.6.A.1, 4.6.A.2, 4.6.A.3**
- How can we represent a curve in the plane parametrically? **4.7.A.1, 4.7.A.2**
- How can we represent a conic section parametrically? **4.7.B.1, 4.7.B.2, 4.7.B.3**

***The indicators listed above correspond to the *AP Precalculus Course and Exam Booklet*.

Reflective

- Why might it be advantageous to describe motion using parametric relationships?
- What justification is needed for a conclusion reached by describing parametric motion?
- Why might it be advantageous to describe conic sections using parametric relationships?

FOCUS STANDARDS

Standards of Mathematical Practice

1.A. Solve equations and inequalities represented analytically, with and without technology.
1.B. Express functions, equations, or expressions in analytically equivalent forms that are useful in a given mathematical or applied context.
1.C. Construct new functions, using transformations, compositions, inverses, or regressions, that may be useful in modeling contexts, criteria, or data, with and without technology.
2.A. Identify information from graphical, numerical, analytical, and verbal representations to answer a question or construct a model, with and without technology.
2.B. Construct equivalent graphical, numerical, analytical, and verbal representations of functions that are useful in a given mathematical or applied context, with and without technology.
3.A Describe the characteristics of a function with varying levels of precision, depending on the function representation and available mathematical tools.
3.B Apply numerical results in a given mathematical or applied context.
3.C Support conclusions or choices with a logical rationale or appropriate data.

Content Standards
None

UNIT 4B - Functions Involving Vectors and Matrices

[NOTE: College Board has designated this as an optional unit; this unit is not tested on the AP exam. However, there are a few topics that must be covered for JCCC credit.]

ESSENTIAL QUESTION	BIG IDEAS
How do vectors and matrices help us describe the world?	Students will describe planar motion using vector-valued functions. Students will apply matrices to model future and past states.

GUIDING QUESTIONS

Content and Process

- What are the characteristics of a vector? **4.8.A.1, 4.8.A.2, 4.8.A.3, 4.8.A.4**

- How are sums and products of vectors, unit vectors, magnitudes of vectors and angle measures between vectors computed? **4.8.B.1, 4.8.B.2, 4.8.B.3, 4.8.C.1, 4.8.C.2, 4.8.D.1, 4.8.D.2**
- How is planar motion represented in terms of vector-valued functions? **4.9.A.1, 4.9.A.2**
- How is the product of two matrices computed? **4.10.A.1, 4.10.A.2**
- How is the inverse of a 2x2 matrix computed? **4.11.A.1, 4.11.A.2, 4.11.A.3, 4.11.A.4**
- How do we apply the value of the determinant to invertibility and vectors? **4.11.B.1, 4.11.B.2, 4.11.B.3**
- How do we determine the output vectors of a linear transformation with a 2x2 matrix? **4.12.A.1, 4.12.A.2, 4.12.A.3, 4.12.A.4, 4.12.A.5**
- How do we determine the association between a linear transformation and a matrix? **4.13.A.1, 4.13.A.2, 4.13.A.3, 4.13.A.4**
- How do we determine the composition of two linear transformations? **4.13.B.1, 4.13.B.2**
- How is the inverse of a linear transformation determined? **4.13.C.1, 4.13.C.2**
- How do we construct a model of a scenario involving transitions between two states using matrices? **4.14.A.1**
- How do we apply matrix models to predict future and past states for *n* transition steps? **4.14.B.1, 4.14.B.2, 4.14.B.3**

***The indicators listed above correspond to the *AP Precalculus Course and Exam Booklet*.

Reflective

- When can inverses of matrices and the product of matrices be computed?
- Why is a linear transformation a function?

FOCUS STANDARDS

Standards of Mathematical Practice

1.B. Express functions, equations, or expressions in analytically equivalent forms that are useful in a given mathematical or applied context.
1.C. Construct new functions, using transformations, compositions, inverses, or regressions, that may be useful in modeling contexts, criteria, or data, with and without technology.
2.A. Identify information from graphical, numerical, analytical, and verbal representations to answer a question or construct a model, with and without technology.
3.A Describe the characteristics of a function with varying levels of precision, depending on the function representation and available mathematical tools.
3.B Apply numerical results in a given mathematical or applied context.
3.C Support conclusions or choices with a logical rationale or appropriate data.

Content Standards

N.VM.1 - Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (*eg*., *v*, |*v*|, ||<u>v</u>||, *v*).

N.VM.2 - Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.
N.VM.3 - Solve problems involving velocity and other quantities that can be represented by vectors.
N.VM.4 - Add and subtract vectors.
N.VM.5 - Multiply a vector by a scalar.
N.VM.6 - Use matrices to represent and manipulate data, *(e.g. representing information in a linear programming problem as a matrix or rewriting a system of equations as a matrix.)*
N.VM.7 - Multiply matrices by scalars to produce new matrices, *(e.g. as when all of the payoffs in a game are doubled.)*
N.VM.8 - Add, subtract, and multiply matrices of appropriate dimensions; find determinants of 2 X 2 matrices.
N.VM.9 - Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.
N.VM.10 - Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.
N.VM.11 - Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.
N.VM.12 - Work with 2 X 2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.